Cognitive Thinking: An International Journal of Interdisciplinary Studies
Vol. 2, Issue-2 (April-June, 2026), pp.356-363, ISSN: 3107-5088, www.cognitivethinking.in

An Intuitionistic Fuzzy Approach to Modelling Complex Systems Under Incomplete Information

Arpika Kaushik
Research Scholar, Department of Mathematics, CCSU Meerut, U.P.
Email: arpikakaushik01@gmail.com

Article: Received: 18/06/2026, Accepted: 25/06/2026, Published:30/06/2026.
D.O.I. https://doi.org/10.5281/zenodo.21395487

Abstract
Complex systems composed of interacting subsystems — such as cyber-physical, industrial, and infrastructure systems — are frequently assessed under conditions where expert knowledge is incomplete, partially withheld, or simply unavailable for certain components. Ordinary fuzzy sets, which assign a single membership degree to each judgement, cannot explicitly represent the hesitation that arises when an expert is unwilling or unable to commit to a definite assessment. This paper develops an intuitionistic fuzzy set (IFS) based modelling framework for risk and criticality assessment of complex systems under incomplete information, in which each expert judgement is represented by a membership degree, a non-membership degree, and a resulting hesitancy (indeterminacy) degree. Linguistic risk assessments are converted into intuitionistic fuzzy numbers (IFNs) and combined using the intuitionistic fuzzy weighted averaging (IFWA) operator, with a weight-renormalisation procedure that formally accommodates missing expert judgements rather than requiring imputation or discarding incomplete records. The framework is demonstrated on a four-subsystem cyber-physical system — a supervisory control and data acquisition (SCADA) architecture — evaluated by three domain experts, one of whom could not assess two of the four subsystems. The resulting aggregated intuitionistic fuzzy risk profiles are ranked using the score and accuracy functions, yielding a complete criticality ranking together with an explicit, quantified measure of residual uncertainty for each subsystem. The results show that the intuitionistic fuzzy approach preserves and communicates the additional uncertainty introduced by incomplete information, rather than concealing it, offering a more transparent basis for risk-informed decision-making than either crisp scoring or ordinary fuzzy modelling.

Keywords: *Intuitionistic fuzzy sets; Incomplete information; Complex systems modelling; Hesitancy degree; Intuitionistic fuzzy weighted averaging; Risk assessment; Cyber-physical systems*

1. Introduction
Complex systems — power grids, industrial control systems, supply networks, and cyber-physical infrastructures — are characterised by numerous interacting subsystems whose overall behaviour cannot be understood by examining components in isolation. Assessing the risk, reliability, or criticality of such systems typically relies on expert judgement, since complete, objective, and continuously updated data on every subsystem is rarely available in practice. Experts may lack sufficient information about a particular component, may disagree with one another, or may

deliberately abstain from judgement when they are not confident, all of which constitutes a form of incomplete information that a modelling framework must be able to represent honestly rather than paper over.

Ordinary fuzzy sets, as introduced by Zadeh (1965), represent imprecision through a single membership degree $\mu \in [0, 1]$, implicitly treating the non-membership degree as the complement $1 - \mu$. This construction cannot distinguish between an expert who actively believes a subsystem is safe (low risk, high confidence) and one who simply does not know (low risk score by default, but with no real confidence behind it). Atanassov (1986) addressed this limitation by introducing intuitionistic fuzzy sets (IFS), in which each element is assigned an independent membership degree μ, non-membership degree ν, and a derived hesitancy degree $\pi = 1 - \mu - \nu$ that explicitly quantifies the indeterminacy left over after membership and non-membership are accounted for. This third degree of freedom makes IFS a natural candidate for modelling complex systems whose assessment is affected by incomplete or partially withheld expert information.

This paper develops and demonstrates an IFS-based modelling framework specifically oriented toward complex systems assessment under incomplete information. Rather than treating missing expert judgements as a data-cleaning problem to be resolved by imputation or exclusion, the framework treats incompleteness as a first-class modelling concern: missing assessments are handled through principled weight renormalisation within the intuitionistic fuzzy weighted averaging (IFWA) aggregation operator, and the resulting hesitancy degree of each aggregated judgement is reported alongside the risk ranking itself, giving decision-makers visibility into how much of the final result rests on solid consensus versus residual uncertainty.

The contributions of this paper are: (1) a structured IFS-based framework for aggregating multi-expert linguistic assessments of complex system components while formally accommodating incomplete (missing) judgements; (2) a demonstration of the framework on a four-subsystem cyber-physical system case study, including a scenario in which one expert cannot assess two of the four subsystems; and (3) an analysis of the resulting hesitancy degrees and their implications for risk-informed decision-making, together with a comparison against a defuzzified crisp baseline.

The remainder of the paper proceeds as follows. Section 2 reviews the literature on intuitionistic fuzzy sets and their application to complex systems. Section 3 presents the mathematical preliminaries of IFS. Section 4 describes the proposed modelling framework. Section 5 presents the case study and numerical results. Section 6 discusses the findings, and Section 7 concludes.

2. Literature Review

Atanassov (1986) introduced intuitionistic fuzzy sets as a generalisation of Zadeh's (1965) fuzzy sets, defining an IFS A on a universe X by the triple $(x, \mu_A(x), \nu_A(x))$ subject to $\mu_A(x) + \nu_A(x) \leq 1$, with the hesitancy degree $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ capturing the residual indeterminacy. Atanassov and Gargov (1989) subsequently extended the concept to interval-valued intuitionistic fuzzy sets, further broadening the representation of uncertainty for situations where even the membership and non-membership degrees cannot be stated as single numbers.

Aggregation of intuitionistic fuzzy information has been extensively studied. Xu (2007) formally defined the intuitionistic fuzzy weighted averaging (IFWA) and intuitionistic fuzzy ordered weighted averaging (IFOWA) operators, together with score and accuracy functions that allow aggregated intuitionistic fuzzy numbers to be ranked in a manner analogous to how real numbers are compared. Xu and Yager (2006) explored geometric aggregation operators for intuitionistic fuzzy information, offering an alternative to averaging-based aggregation for contexts where multiplicative combination of judgements is preferred. Szmidt and Kacprzyk (2000) developed distance and similarity measures between intuitionistic fuzzy sets, which have subsequently been used both for consensus measurement among experts and for ranking alternatives in intuitionistic fuzzy MCDM methods.

Application-oriented work has used IFS extensively in risk and reliability analysis. Studies applying IFS to failure mode and effects analysis (FMEA) have shown that representing severity, occurrence, and detectability judgements as intuitionistic fuzzy numbers, rather than crisp risk priority numbers, better preserves disagreement and uncertainty among cross-functional expert teams, which is precisely the situation in which classical crisp FMEA is most frequently criticised for producing misleadingly precise rankings. Boran, Genç, Kurt, and Akay (2009) applied an intuitionistic fuzzy TOPSIS method to supplier selection, demonstrating the practical viability of combining IFS representation with classical MCDM ranking procedures, an approach conceptually related to the framework developed in this paper.

A comparatively smaller body of work has addressed the specific problem of incomplete or missing expert judgements within an intuitionistic fuzzy framework, as opposed to general uncertainty in judgements that are nonetheless fully provided. Where this has been addressed, it is typically handled either by treating a missing judgement as maximal hesitancy ($\mu = \nu = 0, \pi = 1$) or, as adopted in the present paper, by renormalising aggregation weights among the experts who did provide a judgement. Yager and Xu (2013) and related work on Pythagorean and generalised orthopair fuzzy sets extend the admissible region beyond $\mu + \nu \leq 1$, offering further flexibility for representing extreme judgement combinations, though at increased computational and conceptual complexity relative to the intuitionistic formulation adopted here.

The present paper builds on this literature by applying IFS and the IFWA aggregation operator specifically to the problem of complex-system risk assessment where incomplete expert information is not an edge case to be cleaned away but a structural feature of the problem, and by explicitly reporting the resulting hesitancy degree as part of the decision output rather than only the point-ranking of alternatives.

3. Preliminaries: Intuitionistic Fuzzy Sets

3.1 Definition

Let X be a non-empty universe of discourse. An intuitionistic fuzzy set (IFS) A on X, as defined by Atanassov (1986), is an object of the form:

$$A = \{ \langle x, \mu\_A(x), \nu\_A(x) \rangle : x \in X \}, \text{ where } \mu\_A, \nu\_A : X \to [0, 1] \text{ and } 0 \leq \mu\_A(x) + \nu\_A(x) \leq 1$$

Here $\mu\_A(x)$ denotes the degree of membership and $\nu\_A(x)$ the degree of non-membership of x in A. The hesitancy (or indeterminacy) degree $\pi\_A(x)$ is defined as:

$$\pi\_A(x) = 1 - \mu\_A(x) - \nu\_A(x), \quad \pi\_A(x) \in [0, 1]$$

which represents the portion of judgement that is neither committed to membership nor to non-membership — that is, the degree of hesitation, ignorance, or unavailable information affecting the assessment of x. An ordinary fuzzy set is recovered as the special case $\pi\_A(x) = 0$ for all x, i.e. $\nu\_A(x) = 1 - \mu\_A(x)$.

3.2 Intuitionistic Fuzzy Numbers and Operations

A pair $\alpha = (\mu, \nu)$ satisfying $\mu, \nu \in [0, 1]$ and $\mu + \nu \leq 1$ is termed an intuitionistic fuzzy number (IFN). Given two IFNs $\alpha_1 = (\mu_1, \nu_1)$ and $\alpha_2 = (\mu_2, \nu_2)$, the operations used in this paper are:

$$\alpha_1 \oplus \alpha_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \nu_1\nu_2) \qquad \text{[intuitionistic fuzzy addition]}$$

$$k \cdot \alpha = (1 - (1 - \mu)^k, \nu^k), \quad k > 0 \qquad \text{[scalar multiplication]}$$

3.3 Score and Accuracy Functions

To rank IFNs, Xu (2007) defined the score function $S(\alpha)$ and accuracy function $H(\alpha)$:

$$S(\alpha) = \mu - \nu, \quad S(\alpha) \in [-1, 1] \qquad H(\alpha) = \mu + \nu, \quad H(\alpha) \in [0, 1]$$

Given two IFNs α_1 and α_2, if $S(\alpha_1) > S(\alpha_2)$ then α_1 is regarded as greater than α_2; if $S(\alpha_1) = S(\alpha_2)$, the IFN with the higher accuracy H is regarded as more reliable, since a higher $\mu + \nu$ corresponds to a lower residual hesitancy π. This paper uses S and H, together with the hesitancy degree π, as the primary ranking and uncertainty-reporting mechanism.

3.4 Intuitionistic Fuzzy Weighted Averaging (IFWA)

For a collection of IFNs $\alpha_1, \alpha_2, \ldots, \alpha_n$ with associated weights $w = (w_1, \ldots, w_n)$, $\Sigma wi = 1$, $wi \geq 0$, Xu's (2007) IFWA operator aggregates them into a single IFN as follows:

$$IFWA\_w(\alpha_1, \ldots, \alpha_n) = \langle 1 - \Pi_i(1 - \mu_i)^\wedge w_i,\ \ \Pi_i(\nu_i)^\wedge w_i \rangle$$

The IFWA operator generalises the ordinary weighted average to intuitionistic fuzzy information and is idempotent, bounded, and monotonic. It is the aggregation mechanism adopted for combining multi-expert linguistic assessments in the framework presented in Section 4.

Table 1. Linguistic Risk Scale and Corresponding Intuitionistic Fuzzy Numbers

Linguistic Risk Term	Abbreviation	μ (membership)	ν (non-membership)
Very Low	VL	0.10	0.80
Low	L	0.30	0.60
Medium	M	0.50	0.40
High	H	0.70	0.20
Very High	VH	0.85	0.10

4. Research Methodology

The proposed framework comprises five stages, designed specifically to accommodate the possibility that not every expert can, or will, provide a judgement on every system component.

Step 1 — System decomposition.

The complex system under study is decomposed into its constituent subsystems or components, S = {S1, S2, …, Sm}, each of which will receive an independent risk or criticality assessment. This decomposition is essential for complex systems, where aggregate, whole-system judgements tend to obscure which specific components drive overall risk.

Step 2 — Linguistic elicitation with an explicit abstention option.

A panel of K experts, each assigned a credibility weight wk (Σwk = 1) reflecting seniority, domain relevance, or track record, assesses each subsystem using the linguistic risk scale in Table 1. Crucially, experts are explicitly permitted to abstain from assessing a given subsystem if they judge themselves insufficiently informed, rather than being forced to provide a possibly unreliable numeric guess. This design choice is what allows incompleteness to be represented structurally rather than statistically smoothed over.

Step 3 — Conversion and weight renormalisation.

Each provided linguistic judgement is converted to its corresponding IFN using Table 1. For a given subsystem Sj, let $K(j) \subseteq \{1, \ldots, K\}$ denote the subset of experts who actually provided a judgement. The weights of the responding experts are renormalised so that they sum to one over the available subset:

$$w'k = wk \,/\, \Sigma\ell \in K(j)\ w\ell, \quad for\ k \in K(j)$$

This renormalisation preserves the relative credibility ranking of the responding experts while ensuring the aggregation step remains well defined regardless of how many experts abstained, without requiring an arbitrary imputed value to stand in for missing judgement.

Step 4 — Aggregation.

The available, reweighted expert judgements for each subsystem are aggregated into a single IFN using the IFWA operator defined in Section 3.4, producing an aggregated risk profile $\langle \mu_j, \nu_j \rangle$ and an associated hesitancy degree $\pi_j = 1 - \mu_j - \nu_j$ for every subsystem S_j.

Step 5 — Ranking and uncertainty reporting.

Subsystems are ranked by their score function $S(\alpha_j) = \mu_j - \nu_j$, with ties broken by the accuracy function $H(\alpha_j) = \mu_j + \nu_j$. Unlike conventional crisp or purely fuzzy risk rankings, the framework reports the hesitancy degree π_j alongside each ranked subsystem, giving decision-makers a direct, quantified indication of how much residual uncertainty — arising from disagreement or from incomplete expert participation — underlies each risk estimate.

5. Case Study: Risk Assessment of a Cyber-Physical System Under Incomplete Information

The framework is illustrated using a supervisory control and data acquisition (SCADA) system representative of an industrial cyber-physical system, decomposed into four subsystems: the Sensor Subsystem (FM1), the Power Supply Subsystem (FM2), the Communication Subsystem (FM3), and the Control Actuator Subsystem (FM4). Three domain experts (E1, E2, E3), assigned credibility weights of 0.40, 0.35, and 0.25 respectively based on their seniority and direct familiarity with the installation, were asked to assess the overall risk level of each subsystem using the linguistic scale in Table 1.

Expert E3 indicated that they lacked sufficient first-hand information to responsibly assess the Power Supply Subsystem and the Control Actuator Subsystem, and abstained from providing a judgement on those two components — an explicit instance of incomplete information of the kind the framework is designed to accommodate. The full set of elicited judgements is summarised in Table 2.

Table 2. Expert Linguistic Risk Judgements per Subsystem (— = abstained, incomplete information)

Subsystem	E1 (w=0.40)	E2 (w=0.35)	E3 (w=0.25)	Renormalised Weights Used
FM1 – Sensor Subsystem	H	M	H	0.40 / 0.35 / 0.25
FM2 – Power Supply Subsystem	VH	H	— (abstained)	0.533 / 0.467 / —
FM3 – Communication Subsystem	M	M	L	0.40 / 0.35 / 0.25
FM4 – Control Actuator Subsystem	H	VH	— (abstained)	0.533 / 0.467 / —

5.1 Aggregated Intuitionistic Fuzzy Risk Profiles

Applying the IFWA operator of Step 4 (Section 4) to the (renormalised, where applicable) expert judgements in Table 2 produces the aggregated intuitionistic fuzzy risk profile ⟨μ, ν, π⟩ for each subsystem, shown in Table 3, together with the corresponding score S and accuracy H values.

Table 3. Aggregated Intuitionistic Fuzzy Risk Profiles (IFWA)

Subsystem	⟨μ, ν, π⟩	Score $S = \mu - \nu$	Accuracy $H = \mu + \nu$
Sensor Subsystem	⟨0.641, 0.255, 0.104⟩	0.386	0.896
Power Supply Subsystem	⟨0.793, 0.138, 0.069⟩	0.655	0.931
Communication Subsystem	⟨0.456, 0.443, 0.101⟩	0.013	0.899
Control Actuator Subsystem	⟨0.783, 0.145, 0.072⟩	0.638	0.928

5.2 Ranking

Ranking the four subsystems by score function S (Table 4) yields the following overall criticality ordering, from highest to lowest risk:

Table 4. Final Criticality Ranking with Hesitancy Degree

Rank	Subsystem	Score S	Hesitancy π
1	Power Supply Subsystem	0.655	0.069
2	Control Actuator Subsystem	0.638	0.072
3	Sensor Subsystem	0.386	0.104
4	Communication Subsystem	0.013	0.101

The Power Supply Subsystem (FM2) is identified as the most critical component ($S = 0.655$), followed by the Control Actuator Subsystem (FM4, $S = 0.638$), the Sensor Subsystem (FM1, $S = 0.386$), and finally the Communication Subsystem (FM3, $S = 0.013$), whose near-zero score reflects genuine disagreement among the experts (M, M, L) rather than a confident low-risk assessment.

Notably, the two subsystems affected by an expert abstention — FM2 and FM4 — exhibit the lowest hesitancy degrees in the study ($\pi = 0.069$ and $\pi = 0.072$ respectively), because the two remaining experts happened to agree closely on both components; the weight-renormalisation procedure allowed their consensus to be expressed with full confidence rather than being artificially diluted by a forced third judgement. This illustrates a key property of the proposed treatment of incompleteness: abstention reduces the amount of evidence contributing to a judgement, but does not, by construction, inflate uncertainty when the remaining evidence is itself consistent. By contrast, FM3 shows the least decisive score despite having judgements from all three experts, because those judgements genuinely disagreed (M, M, L) — correctly attributing its uncertainty to substantive disagreement rather than to missing data.

6. Discussion

6.1 Distinguishing Disagreement from Missing Information

A central finding of the case study is that the intuitionistic fuzzy framework allows two qualitatively different sources of uncertainty — expert disagreement (FM3) and expert non-participation (FM2, FM4) — to be distinguished in the final output rather than collapsed into a single ambiguous risk score. A conventional crisp risk priority number, or an ordinary fuzzy score without a hesitancy component, would report only a point estimate for each subsystem and would not reveal that FM3's middling score arises from genuine three-way disagreement while FM2 and FM4's higher scores rest on only two, mutually consistent, expert opinions.

6.2 Comparison with Crisp and Imputation-Based Baselines

For comparison, a crisp baseline was constructed by defuzzifying each linguistic term to its numeric risk level (VL=1 … VH=5) and computing a simple weighted average, imputing a neutral "Medium" rating for E3's two abstentions rather than renormalising weights. This crisp-with-imputation approach produced the same top-ranked subsystem (FM2) but understated its criticality relative to the IFWA result, because the imputed neutral value for the missing E3 judgement pulled the average toward the middle of the scale even though E3 had no actual opinion to contribute. This comparison illustrates a general risk of imputation-based handling of missing data in risk assessment: it can silently bias results toward whatever value is chosen as the default, whereas the weight-renormalisation approach adopted here lets the available evidence speak with its full, undiluted weight.

6.3 Practical Implications

For safety- and security-critical complex systems, the ability to report not just a criticality ranking but also a calibrated indication of how much of that ranking rests on solid multi-expert consensus versus limited or conflicting evidence is directly actionable: subsystems with high risk scores and low hesitancy (such as FM2 in this study) warrant immediate mitigation, whereas subsystems with high hesitancy — whether from disagreement or incomplete participation — may instead warrant a targeted data-gathering or elicitation exercise before a firm mitigation decision is made.

6.4 Limitations

The framework assumes that expert credibility weights are known and fixed in advance; in practice these weights may themselves be uncertain or contested. The linguistic-to-IFN conversion scale in Table 1 is a modelling choice, consistent with common practice in the literature, but alternative scales could shift absolute score values, though the relative ranking behaviour illustrated here is expected to be robust to modest changes in the scale. Finally, the case study involves a relatively small panel of three experts; the weight-renormalisation mechanism becomes increasingly influential, and should be applied with greater caution, as the proportion of abstaining experts grows large relative to the panel size.

7. Conclusion and Future Work

This paper presented an intuitionistic fuzzy set-based framework for modelling complex systems under incomplete information, in which expert risk judgements are represented as intuitionistic fuzzy numbers, aggregated via the IFWA operator with a weight-renormalisation mechanism that formally accommodates missing assessments, and ranked using the score and accuracy functions. Applied to a four-subsystem cyber-physical system case study in which one of three experts could not assess two subsystems, the framework produced a complete criticality ranking together with subsystem-specific hesitancy degrees that correctly distinguished uncertainty arising from expert disagreement from uncertainty arising from incomplete participation — a distinction that crisp and imputation-based approaches were shown to obscure.

Future work includes extending the framework to interval-valued or Pythagorean intuitionistic fuzzy sets for problems in which even the membership and non-membership degrees cannot be stated precisely; incorporating dynamic, time-varying expert weights as new information becomes available; combining the present risk-ranking stage with an intuitionistic fuzzy analytic hierarchy process for deriving subsystem importance weights directly from pairwise comparisons; and validating the framework on a larger, operational dataset with a bigger expert panel to more rigorously characterise the sensitivity of the weight-renormalisation mechanism to high rates of abstention.

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