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Fuzzy Set-Based Modelling of Uncertainty in Multi-Criteria Decision-Making Systems

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Abstract: Real-world decision-making problems are frequently characterised by imprecision, vagueness, and subjectivity that classical crisp models cannot adequately capture. This paper presents a fuzzy set-based modelling framework for multi-criteria decision-making (MCDM) under uncertainty, in which linguistic assessments of decision-makers are represented as triangular fuzzy numbers (TFNs) and aggregated through a fuzzy extension of the Technique for Order Preference by Similarity to Ideal Solution (Fuzzy TOPSIS). The proposed methodology integrates fuzzy set aggregation, fuzzy-weighted normalisation, and vertex-based distance measures to rank competing alternatives while preserving the uncertainty inherent in expert judgement throughout the computational process, rather than eliminating it at the input stage. The framework is illustrated through a supplier-selection case study involving four alternatives evaluated by three decision-makers against four criteria (cost, quality, delivery, and service). The resulting closeness coefficients produce a complete ranking of alternatives, and a discussion of sensitivity and robustness is provided. The results demonstrate that fuzzy set-based modelling offers a transparent, mathematically tractable, and practically implementable approach for group decision-making under uncertainty, with applicability extending to supplier selection, project evaluation, risk assessment, and other domains where qualitative judgement must be converted into defensible quantitative rankings.
Keywords: *Fuzzy sets; Uncertainty modelling; Multi-criteria decision-making; Fuzzy TOPSIS; Triangular fuzzy numbers; Linguistic variables; Supplier selection*

1. Introduction: Decision-making in engineering, management, and the social sciences rarely occurs under conditions of complete information. Decision-makers must routinely evaluate alternatives against multiple, often conflicting, criteria using data that is incomplete, subjective, or expressed in natural language rather than precise numbers. Classical multi-criteria decision-making (MCDM) techniques such as the Analytic Hierarchy Process (AHP) and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) were originally developed for crisp, deterministic data and therefore struggle to represent the vagueness that characterises expert judgement.

Fuzzy set theory, introduced by Zadeh, provides a mathematically rigorous mechanism for representing this type of imprecision by allowing partial membership of an element in a set, in contrast to the binary membership of classical (crisp) sets. Since its introduction, fuzzy set theory has been extended and combined with MCDM techniques to produce a broad family of fuzzy MCDM methods, including fuzzy AHP, fuzzy TOPSIS, fuzzy VIKOR, and fuzzy ELECTRE, each capable of processing linguistic assessments while retaining the interpretability of the original crisp method.

This paper focuses specifically on modelling uncertainty in group MCDM problems using triangular fuzzy numbers (TFNs) and a fuzzy extension of TOPSIS. The motivation for this choice is threefold: (i) TFNs offer a favourable balance between computational simplicity and expressive power for representing linguistic terms; (ii) Fuzzy TOPSIS retains the intuitive geometric interpretation of the crisp method, ranking alternatives by their relative closeness to a fuzzy positive ideal solution (FPIS) and a fuzzy negative ideal solution (FNIS); and (iii) the approach naturally accommodates multiple decision-makers, which is essential in most real organisational settings.

The specific contributions of this paper are as follows: (1) a formal fuzzy set-based modelling framework for uncertainty in group MCDM is presented, integrating linguistic-to-TFN conversion, fuzzy aggregation, fuzzy-weighted normalisation, and vertex distance-based ranking; (2) the framework is applied to a realistic supplier-selection case study involving three decision-makers, four alternatives, and four evaluation criteria; and (3) the sensitivity of the resulting ranking to criteria weights is examined and discussed, together with the limitations and possible extensions of the model.

The remainder of the paper is organised as follows. Section 2 reviews related literature on fuzzy set theory and fuzzy MCDM. Section 3 presents the mathematical preliminaries of fuzzy sets, triangular fuzzy numbers, and linguistic variables. Section 4 describes the proposed fuzzy TOPSIS-based modelling framework. Section 5 presents the case study and numerical results. Section 6 discusses the findings, and Section 7 concludes the paper and outlines directions for future research.

2. Literature Review: The foundational concept of fuzzy sets was introduced by Zadeh (1965), who proposed representing an element's membership in a set as a value in the interval [0, 1] rather than a binary 0 or 1, thereby providing a formal language for reasoning about vagueness. Bellman and Zadeh (1970) subsequently extended this idea to decision-making under fuzzy environments, laying the theoretical groundwork for fuzzy decision analysis. Zadeh (1975) further introduced the concept of a linguistic variable, enabling qualitative terms such as "good" or "poor" to be formally represented and manipulated as fuzzy sets, which underpins most subsequent fuzzy MCDM research.

Building on this foundation, Hwang and Yoon (1981) developed the classical TOPSIS method, which ranks alternatives according to their distance from an ideal and an anti-ideal solution. Chen (2000) extended TOPSIS to a fuzzy environment by representing ratings and weights as triangular fuzzy numbers and defining a vertex distance method to compute the separation of each alternative from the fuzzy positive and negative ideal solutions; this Fuzzy TOPSIS formulation remains one of the most widely applied fuzzy MCDM techniques and forms the methodological basis of the present paper. In parallel, Chang (1996) proposed an extent-analysis method for fuzzy AHP, offering an alternative route for deriving fuzzy priority weights from pairwise comparison matrices.

Beyond ordinary fuzzy sets, several generalisations have been proposed to capture additional dimensions of uncertainty. Atanassov (1986) introduced intuitionistic fuzzy sets, which incorporate both a membership and a non-membership degree, allowing hesitation to be modelled explicitly. Yager (2013) proposed Pythagorean fuzzy sets, relaxing the constraint that membership and non-membership degrees sum to at most one, thereby widening the space of admissible judgements.

Type-2 fuzzy sets, whose membership grades are themselves fuzzy, have similarly been explored for problems where even the membership function is uncertain.

Reviews of the field, such as that of Kahraman (2008) and the extensive two-decade survey by Mardani, Jusoh, and Zavadskas (2015), document a substantial and growing body of work applying fuzzy MCDM methods to supplier selection, project evaluation, healthcare, energy planning, and risk assessment. A recurring conclusion across this literature is that hybrid approaches combining fuzzy weighting methods (such as fuzzy AHP or fuzzy BWM) with fuzzy ranking methods (such as fuzzy TOPSIS or fuzzy VIKOR) tend to outperform single-method approaches in terms of robustness, though at the cost of increased computational and cognitive burden on decision-makers.

The present paper situates itself within this stream of work by combining linguistic-variable-based fuzzy weighting with Chen's (2000) Fuzzy TOPSIS ranking procedure, applying the resulting framework to a realistic group decision-making scenario and examining its behaviour under a concrete numerical case study, which is a gap left comparatively under-illustrated by purely theoretical treatments in the reviewed literature.

3. Preliminaries: Fuzzy Sets and Triangular Fuzzy Numbers

3.1 Fuzzy Sets

A fuzzy set A defined on a universe of discourse X is characterised by a membership function $\mu\_A: X \rightarrow [0, 1]$, where $\mu A(x)$ denotes the degree to which element x belongs to A. Unlike a classical (crisp) set, in which membership is restricted to {0, 1}, a fuzzy set permits partial and graded membership, which makes it a natural representation for concepts such as "high quality" or "low cost" that do not admit sharp boundaries.

3.2 Triangular Fuzzy Numbers

A triangular fuzzy number (TFN), denoted $A = (a, b, c)$ with $a \leq b \leq c$, is a special fuzzy set on the real line whose membership function is defined as:

$\mu A(x) = 0 \text{ for } x < a;\ \ (x - a) / (b - a) \text{ for } a \leq x \leq b;\ \ (c - x) / (c - b) \text{ for } b \leq x \leq c;\ \ 0 \text{ for } x > c$

Here, b is the most likely (modal) value, while a and c represent the lower and upper bounds of plausible values, respectively. Given two TFNs $A = (a1, b1, c1)$ and $B = (a2, b2, c2)$, the arithmetic operations used in this paper are defined element-wise as follows:

$A \oplus B = (a1 + a2, b1 + b2, c1 + c2)$ *[fuzzy addition]*

$A \otimes B \approx (a1a2, b1b2, c1c2)$ *[fuzzy multiplication, approximate]*

$d(A, B) = \sqrt{[(1/3)\{(a1 - a2)^2 + (b1 - b2)^2 + (c1 - c2)^2\}]}$ *[vertex distance]*

The vertex distance measure d(A, B), introduced by Chen (2000), provides a crisp scalar distance between two triangular fuzzy numbers and is the central metric used to compute how far each alternative lies from the fuzzy ideal solutions in Section 4.

3.3 Linguistic Variables

A linguistic variable takes words or sentences from natural language as its values, each of which is subsequently represented by a corresponding TFN. This paper uses the seven-point linguistic rating scale and six-point importance-weighting scale summarised in Table 1 and Table 2, which are consistent with the scales commonly adopted in the fuzzy MCDM literature (Chen, 2000; Kahraman, 2008.

Table 1. Linguistic Scale for Rating Alternatives

Linguistic Term	Abbreviation	Triangular Fuzzy Number
Very Poor	VP	(0, 0, 1)
Poor	P	(0, 1, 3)
Medium Poor	MP	(1, 3, 5)
air	F	(3, 5, 7)
Medium Good	MG	(5, 7, 9)
Good	G	(7, 9, 10)
Very Good	VG	(9, 10, 10)

Table 2. Linguistic Scale for Criteria Weights

Linguistic Term	Abbreviation	Triangular Fuzzy Number
Low	L	(0, 0, 0.3)
Medium Low	ML	(0, 0.3, 0.5)
Medium	M	(0.3, 0.5, 0.7)
Medium High	MH	(0.5, 0.7, 0.9)
High	H	(0.7, 0.9, 1)
Very High	VH	(0.9, 1, 1)

4. Research Methodology

The proposed framework models uncertainty in group MCDM through six sequential stages, described below and applied in Section 5.

Step 1 — Problem structuring.

Identify the set of alternatives A = {A1, A2, …, Am}, the set of evaluation criteria C = {C1, C2, …, Cn}, and the panel of K decision-makers. Classify each criterion as benefit-type (higher is better) or cost-type (lower is better).

Step 2 — Linguistic assessment.

Each decision-maker rates every alternative against every criterion using the linguistic scale in Table 1, and rates the relative importance of every criterion using the linguistic scale in Table 2. This step captures subjective judgment directly in natural language rather than forcing premature numerical precision.

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Step 3 — Aggregation.
Individual linguistic ratings are converted to TFNs and aggregated across the K decision-makers by averaging corresponding components, producing an aggregated fuzzy decision matrix $\tilde{X} = [\tilde{x}ij]$ and an aggregated fuzzy weight vector $\tilde{w}j$ for each criterion:

$$\tilde{x}ij = (1/K) \otimes [\tilde{x}ij(1) \oplus \tilde{x}ij(2) \oplus \ldots \oplus \tilde{x}ij(K)]$$

Step 4 — Normalisation.
The aggregated fuzzy decision matrix is normalised to bring all criteria onto a comparable [0, 1] scale, using linear normalisation that preserves the triangular shape of each fuzzy number. For benefit criteria, each TFN is divided by the largest upper bound across alternatives (c*j); for cost criteria, the smallest lower bound (a*j) is divided by each TFN component:

$$\tilde{r}ij = (aij / c^*j,\ bij / c^*j,\ cij / c^*j) \quad \text{for benefit criteria;} \quad \tilde{r}ij = (a^*j / cij,\ a^*j / bij,\ a^*j / aij) \quad \text{for cost criteria}$$

Step 5 — Weighting and ideal-solution distance.
The normalised matrix is combined with the aggregated fuzzy weights to obtain the weighted normalised fuzzy decision matrix, $\tilde{v}ij = \tilde{r}ij \otimes \tilde{w}j$. Because normalisation bounds every $\tilde{r}ij$ within [0, 1], the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) are taken as $A^* = (1, 1, 1)$ and $A^- = (0, 0, 0)$ for every criterion. The vertex distance of each alternative from A^* and A^- is then computed and summed across all criteria:

$$d{+}i = \Sigma j\ d(\tilde{v}ij, A^*) \qquad d{-}i = \Sigma j\ d(\tilde{v}ij, A^-)$$

Step 6 — Closeness coefficient and ranking.
Each alternative is finally ranked according to its closeness coefficient CC_i which represents its relative proximity to the fuzzy positive ideal solution; alternatives with CC_i closer to 1 are preferred:

$$CC_i = d{-}i / (d{+}i + d{-}i), \quad 0 \le CC_i \le 1$$

5. Case Study: Fuzzy Supplier Selection

The proposed framework is illustrated using a representative supplier-selection problem, a domain in which fuzzy MCDM is extensively applied because supplier attributes such as quality and service are inherently difficult to quantify precisely. A manufacturing firm must select the most suitable supplier from four candidates, A1–A4, evaluated by a panel of three decision-makers (DM1–DM3) against four criteria: cost (C1, cost-type), quality (C2, benefit-type), delivery reliability (C3, benefit-type), and service level (C4, benefit-type).

5.1 Criteria Weights

Each decision-maker independently assigned a linguistic importance rating to each criterion using the scale in Table 2. The individual judgements were aggregated by component-wise averaging to obtain the fuzzy criteria weights shown in Table 3.

Table 3. Linguistic Ratings and Aggregated Fuzzy Weights of Criteria

Criterion	DM1	DM2	DM3	Aggregated Fuzzy Weight
C1 – Cost	H	MH	H	(0.633, 0.833, 0.967)

Criterion	DM1	DM2	DM3	Aggregated Fuzzy Weight
C2 – Quality	VH	H	VH	(0.833, 0.967, 1)
C3 – Delivery	MH	M	MH	(0.433, 0.633, 0.833)
C4 – Service	M	MH	M	(0.367, 0.567, 0.767)

5.2 Aggregated Fuzzy Decision Matrix

Each decision-maker rated the four suppliers against each criterion using the linguistic scale in Table 1. The individual ratings were converted to TFNs and aggregated across the three decision-makers, yielding the aggregated fuzzy decision matrix presented in Table 4.

Table 4. Aggregated Fuzzy Decision Matrix

Alternative	C1 – Cost	C2 – Quality	C3 – Delivery	C4 – Service
A1	(5.667, 7.667, 9.333)	(7.667, 9.333, 10)	(3.667, 5.667, 7.667)	(4.333, 6.333, 8.333)
A2	(3.667, 5.667, 7.667)	(5.667, 7.667, 9.333)	(7.667, 9.333, 10)	(6.333, 8.333, 9.667)
A3	(7.667, 9.333, 10)	(4.333, 6.333, 8.333)	(4.333, 6.333, 8.333)	(3.667, 5.667, 7.667)
A4	(1.667, 3.667, 5.667)	(5.667, 7.667, 9.333)	(5.667, 7.667, 9.333)	(5.667, 7.667, 9.333)

5.3 Normalised and Weighted Normalised Matrices

Table 5. Normalised Fuzzy Decision Matrix

Alternative	C1 – Cost	C2 – Quality	C3 – Delivery	C4 – Service
A1	(0.179, 0.217, 0.294)	(0.767, 0.933, 1)	(0.367, 0.567, 0.767)	(0.448, 0.655, 0.862)
A2	(0.217, 0.294, 0.455)	(0.567, 0.767, 0.933)	(0.767, 0.933, 1)	(0.655, 0.862, 1)
A3	(0.167, 0.179, 0.217)	(0.433, 0.633, 0.833)	(0.433, 0.633, 0.833)	(0.379, 0.586, 0.793)
A4	(0.294, 0.455, 1)	(0.567, 0.767, 0.933)	(0.567, 0.767, 0.933)	(0.586, 0.793, 0.965)

Applying the normalisation procedure of Step 4 (Section 4) to the aggregated matrix in Table 4, with cost criterion C1 normalised using a^*j and benefit criteria C2–C4 normalised using c^*j, produces the normalised fuzzy decision matrix in Table 5. Multiplying each normalised value by its corresponding aggregated criterion weight from Table 3 then yields the weighted normalised fuzzy decision matrix in Table 6.

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Table 6. Weighted Normalised Fuzzy Decision Matrix

Alternative	C1 – Cost	C2 – Quality	C3 – Delivery	C4 – Service
A1	(0.113, 0.181, 0.284)	(0.639, 0.902, 1)	(0.159, 0.359, 0.639)	(0.164, 0.371, 0.661)
A2	(0.137, 0.245, 0.44)	(0.472, 0.742, 0.933)	(0.332, 0.591, 0.833)	(0.24, 0.489, 0.767)
A3	(0.106, 0.149, 0.21)	(0.361, 0.612, 0.833)	(0.187, 0.401, 0.694)	(0.139, 0.332, 0.608)
A4	(0.186, 0.379, 0.967)	(0.472, 0.742, 0.933)	(0.246, 0.486, 0.777)	(0.215, 0.45, 0.74)

5.4 Distance to Ideal Solutions and Final Ranking

With the fuzzy positive and negative ideal solutions set to A* = (1, 1, 1) and A⁻ = (0, 0, 0) for every criterion, the vertex distance d+i and d−i of each alternative was computed by summing the distances across all four criteria. The resulting closeness coefficients CC_i and final ranking are reported in Table 7.

Table 7. Distance to Ideal Solutions, Closeness Coefficients, and Final Ranking

Alternative	d+i (distance to FPIS)	d−i (distance to FNIS)	CC_i	Rank
A1	2.306	1.946	0.458	3
A2	2.086	2.205	0.514	2
A3	2.566	1.676	0.395	4
A4	2.049	2.412	0.541	1

The final ranking obtained is A4 ≻ A2 ≻ A1 ≻ A3, meaning supplier A4 is the most preferred alternative, followed by A2, A1, and finally A3. Although A4 received the least favourable cost ratings among the four suppliers, its consistently strong performance on quality, delivery, and service — combined with the relatively lower weight assigned to service and delivery compared to quality — resulted in the highest overall closeness coefficient (CC = 0.541). Supplier A3, despite the best cost rating, ranked last (CC = 0.395) due to comparatively weaker quality, delivery, and service scores, illustrating how the fuzzy-weighted aggregation prevents any single criterion from dominating the outcome.

6. Results and Discussion

6.1 Sensitivity to Criteria Weights

To assess the robustness of the ranking, the modal (middle) component of the quality weight (C2) was varied from 0.7 to 1.0 in increments of 0.1 while proportionally rescaling the remaining weights, and the closeness coefficients were recomputed at each step. Across this range, A4 and A2 consistently occupied the top two positions, while A3 remained last, indicating that the overall ranking is reasonably stable with respect to moderate variation in the emphasis placed on quality.

This stability is consistent with findings reported elsewhere in the fuzzy MCDM literature (Mardani et al., 2015), where TOPSIS-based rankings are generally found to be less sensitive to weight perturbation than utility-based aggregation methods.

6.2 Comparison with Crisp Modelling

A parallel evaluation using defuzzified (crisp) input data — obtained by replacing each TFN with its centroid value prior to ranking — produced the same ordinal ranking (A4, A2, A1, A3) but collapsed the meaningful spread between closely ranked alternatives A1 and A2 into a narrower, less informative numerical gap. This confirms that while fuzzy modelling and crisp point-estimate modelling can agree on the final ranking, the fuzzy approach retains information about the degree of uncertainty and the closeness between competing alternatives that a purely crisp model discards, which is valuable when the decision is close or contested.

6.3 Managerial and Practical Implications

The case study demonstrates that fuzzy set-based modelling allows decision-makers to express judgements in natural language without being forced into artificial numerical precision, while still producing a fully quantified, defensible ranking. This is particularly valuable in group decision contexts, such as supplier selection, project prioritisation, or personnel evaluation, where individual assessors may have differing scales of judgement that are more naturally reconciled at the linguistic rather than the numerical level.

6.4 Limitations

The framework presented here relies on triangular fuzzy numbers and equal treatment of decision-makers; it does not account for decision-maker-specific confidence weighting, nor does it capture hesitation or non-membership explicitly as intuitionistic or Pythagorean fuzzy extensions would. The linguistic-to-TFN conversion scale, while widely used, is itself a modelling choice that can influence results, and a formal sensitivity analysis of the scale itself is left for future work.

7. Conclusion and Future Work

This paper presented a fuzzy set-based modelling framework for representing and processing uncertainty in multi-criteria group decision-making, combining linguistic variable theory, triangular fuzzy number aggregation, and a fuzzy extension of TOPSIS. The framework was applied to a realistic supplier-selection case study involving four alternatives, four criteria, and three decision-makers, producing a complete and interpretable ranking together with quantified distances to the fuzzy ideal solutions. Sensitivity analysis indicated that the resulting ranking is reasonably robust to moderate changes in criteria weighting, and comparison with a defuzzified crisp baseline confirmed that the fuzzy approach preserves valuable information about the closeness between competing alternatives.

Future research directions include: extending the framework to intuitionistic or Pythagorean fuzzy sets to capture hesitation more explicitly; incorporating decision-maker-specific credibility weighting; combining the present ranking stage with a fuzzy AHP or fuzzy Best-Worst Method weighting stage to reduce reliance on directly elicited linguistic importance judgements; and validating the framework against a larger empirical dataset drawn from an operational organisational setting.

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